Tableau calculi for propositional dependence logics

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(Joint work with Katsuhiko Sano)

Motivation and history

Logical modelling of uncertainty, imperfect information and functional dependence in the framework of modal logic.

The ideas are transfered from first-order dependence logic (and independence-friendly logic) to modal logic.

Historical development:

- Branching quantifiers by Henkin 1959.
- Independence-friendly logic by Hintikka and Sandu 1989.
- Compositional semantics for independence-friendly logic by Hodges 1997. (Origin of team semantics.)
- IF modal logic by Tulenheimo 2003.
- Dependence logic by Väänänen 2007.
- Modal dependence logic by Väänänen 2008.

Standard modal logic

Definition

Let Φ be a set of atomic propositions. The set of formulae for standard modal logic $\mathcal{ML}(\Phi)$ is generated by the following grammar

$$\varphi ::= p \mid \neg p \mid (\varphi \vee \varphi) \mid (\varphi \wedge \varphi) \mid \Diamond\varphi \mid \Box\varphi,$$

where $p \in \Phi$.

Definition

Let Φ be a set of atomic propositions. A Kripke model K over Φ is a tuple

$$K = (W, R, V),$$

where W is a nonempty set of *worlds*, $R \subseteq W \times W$ is a binary relation, and V is a *valuation* $V : \Phi \to \mathcal{P}(W)$.

Semantics for modal logic

Definition

Kripke semantics for $\mathcal{ML}$ is defined as follows.

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\begin{aligned}
K, w \models p &\quad\Leftrightarrow\quad w \in V(p).\\
K, w \models \neg p &\quad\Leftrightarrow\quad w \notin V(p).\\
K, w \models \varphi \vee \psi &\quad\Leftrightarrow\quad K, w \models \varphi \text{ or } K, w \models \psi.\\
K, w \models \varphi \wedge \psi &\quad\Leftrightarrow\quad K, w \models \varphi \text{ and } K, w \models \psi.\\
K, w \models \Diamond\varphi &\quad\Leftrightarrow\quad K, w' \models \varphi, \text{ for some } w' \text{ s.t. } xRw'.\\
K, w \models \Box\varphi &\quad\Leftrightarrow\quad K, w \models \varphi, \text{ for all } w' \text{ s.t. } xRw'.
\end{aligned}
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2. The standard semantics for modal logic is given with respect to pointed models K, w. In team semantics the semantics is given for models and teams, i.e., with respect to pairs K, T, where T is a team of K.

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1. In this context a team is a set of possible worlds, i.e., if $K = (W, R, V)$ is a Kripke model then $T \subseteq W$ is a team of K.
2. The standard semantics for modal logic is given with respect to pointed models K, w. In team semantics the semantics is given for models and teams, i.e., with respect to pairs K, T, where T is a team of K.
3. Some possible interpretations for K, w and K, T:
   (a) $K, w \models \varphi$: The actual world is w and φ is true in w.
   (b) $K, T \models \varphi$: The actual world is in T, but we do not know which one it is. The formula φ is true in the actual world.
   (c) $K, T \models \varphi$: We consider sets of points as primitive. The formula φ describes properties of collections of points.

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Kripke/Team semantics for $\mathcal{ML}$ is defined as follows. Remember that $K = (W, R, V)$ is a normal Kripke model and $T \subseteq W$.

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K, w \models \Box\varphi &\quad\Leftrightarrow\quad K, w' \models \varphi \text{ for every } w' \text{ s.t. } wRw'. \\
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K, T &\models \varphi \wedge \psi && \Leftrightarrow \quad K, T \models \varphi \text{ and } K, T \models \psi.\\
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K, w \models \Box\varphi \quad &\Leftrightarrow \quad K, w' \models \varphi \text{ for every } w' \text{ s.t. } wRw'.\\
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K, T \models \Box\varphi &\quad\Leftrightarrow\quad K, T' \models \varphi \text{ for } T' := \{w' \mid w \in T, wRw'\}.\\
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K, T \models \Diamond\varphi \quad &\Leftrightarrow \quad K, T' \models \varphi \text{ for some } T' \text{ s.t.}\\
&\qquad \forall w \in T\, \exists w' \in T' : wRw' \text{ and } \forall w' \in T'\, \exists w \in T : wRw'.
\end{aligned}
$$

Note that $K, \emptyset \models \varphi$ for every formula φ.

Team semantics vs. Kripke semantics

Theorem (Flatness property of $\mathcal{ML}$)

Let K be a Kripke model, T a team of K and φ a $\mathcal{ML}$-formula. Then

$$K, T \models \varphi \quad \Leftrightarrow \quad K, w \models \varphi \text{ for all } w \in T,$$

in particular

$$K, \{w\} \models \varphi \quad \Leftrightarrow \quad K, w \models \varphi.$$

Note that it also follows that every $\mathcal{ML}$-formula is *downwards closed*:

$$\text{If } K, T \models \varphi, \text{ then } K, S \models \varphi \text{ for all } S \subseteq T.$$

Extended modal dependence logic

Introduced by Ebbing et al. 2013, the syntax of extended modal dependence logic $\mathcal{EMDL}(\Phi)$ extends the syntax of modal logic by the clause

$$\mathrm{dep}(\varphi_1, \ldots, \varphi_n, \psi),$$

where $\varphi_1, \ldots, \varphi_n, \psi$ are formulae of $\mathcal{ML}(\Phi)$.

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The intended meaning of the atomic formula

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The semantics for $\mathcal{EMDL}$ extends the sematics of $\mathcal{ML}$, defined with teams, by the following clause:

$$K, T \models \mathrm{dep}(\varphi_1, \ldots, \varphi_n, \psi)$$

if and only if for every $w_1, w_2 \in T$:

$$\bigwedge_{i \leq n} (K, w_1 \models \varphi_i \Leftrightarrow K, w_2 \models \varphi_i) \Rightarrow (K, w_1 \models \psi \Leftrightarrow K, w_2 \models \psi).$$

Intuitionistic disjunction and expressive power

$\mathcal{ML}(\varovee)$: add a different version of disjunction $\varovee$ to modal logic with the semantics:

- $K, T \models \varphi \varovee \psi \iff K, T \models \varphi$ or $K, T \models \psi$.

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Theorem (Ebbing, Hella, Meier, Müller, V., Vollmer 13)

$\mathcal{EMDL} \leq \mathcal{ML}(\varovee)$.

Theorem (Hella, Luosto, Sano, V. 14)

$\mathcal{ML}(\varovee) \leq \mathcal{EMDL}$.

Thus $\mathcal{EMDL} \equiv \mathcal{ML}(\varovee)$. Furthermore $\mathcal{ML}(\varovee) \equiv \bigvee \mathcal{ML}$.

Modal definability

It is well-known that modal definability can be characterized in terms of closure under k-bisimulation:

Theorem (Gabbay, van Benthem)

A class $\mathcal{C}$ of pointed Kripke models is definable in $\mathcal{ML}$ if and only if $\mathcal{C}$ is closed under k-bisimulation for some $k \in \mathbb{N}$.

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Theorem (Hella, Luosto, Sano, V. 14)

A class $\mathcal{C}$ is definable in $\mathcal{EMDL}$ (in $\mathcal{ML}(\otimes)$) if and only if $\mathcal{C}$ is downward closed and there exists $k \in \mathbb{N}$ such that $\mathcal{C}$ is closed under team k-bisimulation.

Complexity results

Propositional dependence logic ($\mathcal{PD}$) is defined as the modal free fragment of $\mathcal{EMDL}$.

	SAT	VAL
$\mathcal{PL}$	NP [1]	coNP [1]
$\mathcal{ML}$	PSPACE [2]	PSPACE [2]
$\mathcal{PD}$	NP [3]	NEXPTIME [5]
$\mathcal{EMDL}$	NEXPTIME [4]	in $\text{NEXPTIME}^{\text{NP}}$ [5]

[1] Cook 1971, Levin 1973, [2] Ladner 1977, [3] Lohmann, Vollmer 2013,
[4] Ebbing, Hella, Meier, Müller, V., Vollmer 2013, [5] V. 2014.

Towards for a tableau calculus for $\mathcal{EMDL}$

We say that of formula φ is *k-coherent*, $k \in \mathbb{N}$, iff the equivalence

$$K, T \models \varphi \quad \Leftrightarrow \quad K, T' \models \varphi, \text{ for every } T' \subseteq T \text{ s.t } |T'| \leq k$$

holds for every Kripke model K and every team T of K.

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Let φ be a formula of $\mathcal{EMDL}$ ($\mathcal{ML}(\varovee)$). Then φ is $2^{2^{|\varphi|}}$-coherent ($2^{|\varphi|}$-coherent).

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Thus a formula φ of $\mathcal{EMDL}$ or $\mathcal{ML}(\varovee)$ is valid if and only if φ is valid in the class of "small" models.

The intuition behind our labeled tableau calculi

- The expressions that occur in our calculi are labeled formulae, i.e., expressions of the form $\alpha : \varphi$, where $\alpha \subseteq \mathbb{N}$ is a finite set and φ is a formula of some logic.
- The intuitive meaning of $\alpha : \varphi$ is that the team that corresponds to α falsifies φ.

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- Fix a logic $\mathcal{L}$ and a calculus $\mathbf{T}_{\mathcal{L}}$.
  - We say that a tableau $\mathcal{T}$ is a tableau for $\varphi \in \mathcal{L}$ if the root of $\mathcal{T}$ is $\{1, \dots, 2^{2^{|\varphi|}}\} : \varphi$ and $\mathcal{T}$ is obtained from $\{1, \dots, 2^{2^{|\varphi|}}\} : \varphi$ by applying the rules of $\mathbf{T}_{\mathcal{L}}$.

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  - We say that $\varphi \in \mathcal{L}$ is provable in $\mathbf{T}_{\mathcal{L}}$, and write $\vdash_{\mathbf{T}_{\mathcal{L}}} \varphi$, if there exists a closed tableau for φ.

Rules for contradiction

$$\frac{\{i\} : p \quad \{i\} : \neg p}{X}$$

$$\frac{\emptyset : \varphi}{X}$$

$$\frac{\{i\} : \mathrm{dep}(\varphi_1, \ldots, \varphi_n, \psi)}{X}$$

Rules for propositional connectives

$$\frac{\{i_1, \ldots, i_k\} : p}{\{i_1\} : p \mid \ldots \mid \{i_k\} : p} (Prop) \qquad \frac{\{i_1, \ldots, i_k\} : \neg p}{\{i_1\} : \neg p \mid \ldots \mid \{i_k\} : \neg p} (\neg Prop)$$

$$\frac{\alpha : (\varphi \wedge \psi)}{\alpha : \varphi \mid \alpha : \psi} (\wedge) \qquad \frac{\alpha : (\varphi \varovee \psi)}{\begin{array}{c}\alpha : \varphi \\ \alpha : \psi\end{array}} (\varovee)$$

$$\frac{\alpha : (\varphi \vee \psi)}{\beta : \varphi \mid \alpha \setminus \beta : \psi} (\vee) \text{ where } \beta \subseteq \alpha$$

Rules for modalities

$$\frac{\begin{array}{c} i_1 R j_1 \\ \vdots \\ i_n R j_n \\ \{i_1, \dots, i_n\} : \Diamond\varphi \end{array}}{\{j_1, \dots, j_n\} : \varphi} \ (\Diamond)$$

$$\frac{\alpha : \Box\varphi}{\begin{array}{ccccc} f_1(1) R i_1 & \mid & \dots & \mid & f_k(1) R i_1 \\ \vdots & & \vdots & & \vdots \\ f_1(t) R i_t & \mid & \dots & \mid & f_k(t) R i_t \\ \{i_1, \dots i_t\} : \varphi & \mid & \dots & \mid & \{i_1, \dots i_t\} : \varphi \end{array}} \ (\Box)\dagger$$

$\dagger$: $t = 2^{2^{|\varphi|}}$ and $f_1, \dots, f_k$ denote exactly all functions with domain $\{1, \dots, t\}$ and co-domain α, and $i_1, \dots, i_t$ are fresh and distinct.

Rules for dependence atoms

$$\frac{\alpha \,:\, \mathrm{dep}(\varphi_1, \ldots, \varphi_n, \psi)}{\alpha_1 \,:\, \mathrm{dep}(\varphi_1, \ldots, \varphi_n, \psi) \mid \ldots \mid \alpha_k \,:\, \mathrm{dep}(\varphi_1, \ldots, \varphi_n, \psi)} \; (\mathit{Split})\dagger$$

$\dagger$: $\alpha_1, \ldots, \alpha_k$ are exactly all subsets of α of cardinality 2.

$$\frac{\{i_1, i_2\} \,:\, \mathrm{dep}(\varphi_1, \ldots, \varphi_n, \psi)}{\begin{array}{ccc} \{i_1\} : \varphi_1^{h_1(1)} & \mid \ldots \mid & \{i_1\} : \varphi_1^{h_k(1)} \\ \{i_2\} : \varphi_1^{h_1(1)} & \mid \ldots \mid & \{i_2\} : \varphi_1^{h_k(1)} \\ \vdots & \vdots & \vdots \\ \{i_1\} : \varphi_n^{h_1(n)} & \mid \ldots \mid & \{i_1\} : \varphi_n^{h_k(n)} \\ \{i_2\} : \varphi_n^{h_1(n)} & \mid \ldots \mid & \{i_2\} : \varphi_n^{h_1(n)} \\ \{i_1, i_2\} : \psi & \mid \ldots \mid & \{i_1, i_2\} : \psi \\ \{i_1, i_2\} : \psi^{\bot} & \mid \ldots \mid & \{i_1, i_2\} : \psi^{\bot} \end{array}} \; (\mathit{dep})\ddagger$$

$\ddagger$: $h_1, \ldots h_k$ denotes all the functions with domain $\{1, \ldots, n\}$ and co-domain $\{\top, \bot\}$. By $\varphi^{\bot}$ we denote the negation formal form of $\neg\varphi$, and $\varphi^{\top}$ denotes φ.

Results

Let $\mathbf{T}_{\mathcal{ML}} := \{(Prop), (\neg Prop), (\wedge), (\vee), (\Diamond), (\Box)\}$.

Let $\mathbf{T}_{\mathcal{ML}(\varovee)} := \mathbf{T}_{\mathcal{ML}} \cup \{(\varovee)\}$, and $\mathbf{T}_{\mathcal{EMDL}} := \mathbf{T}_{\mathcal{ML}} \cup \{(Split), (dep)\}$.

Theorem (Sano, V. 2015?)

$\mathbf{T}_{\mathcal{ML}}$, $\mathbf{T}_{\mathcal{ML}(\varovee)}$, and $\mathbf{T}_{\mathcal{EMDL}}$ are sound and complete with respect to team semantics of $\mathcal{ML}$, $\mathcal{ML}(\varovee)$, and $\mathcal{EMDL}$, respectively.

We also obtain corresponding results for $\mathcal{PL}$, $\mathcal{PL}(\varovee)$, $\mathcal{PD}$, and $\mathcal{MDL}$ (modal dependence logic).

In addition we obtain Hilbert-style axiomatizations for all of the above logics.

Thanks!

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