Definability in modal logics with team semantics

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Prologue

Modal logic

Set Φ of atomic propositions. The formulae of $\mathcal{ML}(\Phi)$ are generated by:

$$\varphi ::= p \mid \neg p \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid \Diamond\varphi \mid \Box\varphi$$

Semantics via pointed Kripke structures $(W, R, V), w$. Nonempty set W, binary relation $R \subseteq W^2$, valuation $V : \Phi \to \mathcal{P}(W)$, point $w \in W$.

E.g.,

- $K, w \models p$ iff $w \in V(p)$,
- $K, w \models \Diamond\varphi$ iff $K, v \models \varphi$ for some v s.t. wRv.

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Modal logics with team semantics

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Semantics via team-pointed Kripke structures $(W, R, V), T$. Nonempty set W, binary relation $R \subseteq W^2$, valuation $V : \Phi \to \mathcal{P}(W)$, team $T \subseteq W$.

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Modal logics with team semantics

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Logics of interest

Extensions of modal logic with:

- Propositional dependence atoms: $\mathcal{MDL}$
  $\mathrm{dep}(p_1, \ldots, p_n, q)$
- Modal dependence atoms: $\mathcal{EMDL}$
  $\mathrm{dep}(\varphi_1, \ldots, \varphi_n, \psi)$
- Inclusion atoms: $\mathcal{ML}(\subseteq)$
- Intuitionistic disjunction: $\mathcal{ML}(\varovee)$
  $K, T \models \varphi \varovee \psi$ iff $K, T \models \varphi$ or $K, T \models \psi$
- Universal modality: $\mathcal{ML}(\boxed{u})$

Expressive power of modal logics

Theorem (Gabbay, van Benthem)

A class $\mathcal{C}$ of pointed Kripke models is definable in $\mathcal{ML}$ if and only if $\mathcal{C}$ is closed under k-bisimulation for some $k \in \mathbb{N}$.

Theorem (Hella, Stumpf 2015)

A nonempty class $\mathcal{C}$ of team-pointed Kripke models is definable in $\mathcal{ML}(\subseteq)$ if and only if $\mathcal{C}$ is union closed and there exists $k \in \mathbb{N}$ such that $\mathcal{C}$ is closed under team k-bisimulation.

Theorem (Hella, Luosto, Sano, V. 2014)

A nonempty class $\mathcal{C}$ of team-pointed Kripke models is definable in $\mathcal{ML}(\otimes)$ if and only if $\mathcal{C}$ is downward closed and there exists $k \in \mathbb{N}$ such that $\mathcal{C}$ is closed under team k-bisimulation.

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Theorem (Hella, Luosto, Sano, V. 2014)

A nonempty class $\mathcal{C}$ of team-pointed Kripke models is definable in $\mathcal{ML}(\varovee)$ if and only if $\mathcal{C}$ is downward closed and there exists $k \in \mathbb{N}$ such that $\mathcal{C}$ is closed under team k-bisimulation.

Expressive power of modal logics

Theorem (van Benthem's theorem)

A class $\mathcal{C}$ of pointed Kripke models is definable in $\mathcal{ML}$ if and only if $\mathcal{C}$ is definable in $\mathcal{FO}$ and closed under bisimulation.

Via a recent result of Kontinen, Müller, Schnoor, and Vollmer on $\mathcal{ML}(\sim)$:

Corollary

A nonempty class $\mathcal{C}$ of team-pointed Kripke models is definable in $\mathcal{ML}(\subseteq)$ if and only if $\mathcal{C}$ is union closed, definable in $\mathcal{FO}$, and closed under team bisimulation.

Corollary

A nonempty class $\mathcal{C}$ of team-pointed Kripke models is definable in $\mathcal{ML}(\varovee)$ if and only if $\mathcal{C}$ is downward closed, definable in $\mathcal{FO}$, and closed under team bisimulation.

Expressive power of modal logics

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Expressive power

Extended modal dependence logic $\mathcal{EMDL}$:

$K, T \models \mathrm{dep}(\varphi_1, \ldots, \varphi_n, \psi)$ iff $\forall w_1, w_2 \in T$:

$$\bigwedge_{i \leq n} (\{w_1\} \in V(\varphi_i) \Leftrightarrow \{w_2\} \in V(\varphi_i)) \Rightarrow (\{w_1\} \in V(\psi) \Leftrightarrow \{w_2\} \in V(\psi)).$$

Theorem (Hella, Luosto, Sano, V. 2014)

A class of team-pointed Kripke models is definable in $\mathcal{EMDL}$ if and only if it is definable in $\mathcal{ML}(\otimes)$.

Validity in models and frames

- Pointed model (K, w): $(W, R, V), w$
- Model (K): (W, R, V)
- Frame (F): (W, R)

We write:

- $(W, R, V) \models \varphi$ iff $(W, R, V), w \models \varphi$ holds for every $w \in W$
- $(W, R) \models \varphi$ iff $(W, R, V) \models \varphi$ holds for every valuation V

Every (set of) $\mathcal{ML}$-formula defines the class of frames in which it is valid.

- $Fr(\varphi) := \{(W, R) \mid (W, R) \models \varphi\}$.
- $Fr(\Gamma) := \{(W, R) \mid \forall \varphi \in \Gamma : (W, R) \models \varphi\}$.

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Frame definability

Definability

Which properties of graphs can be described with a given logic $\mathcal{L}$.

Example first-order logic on graphs $G = (V, E)$:

- Single formula: $\exists x \exists y \neg x = y$ defines the class $\{(V, E) \mid |V| \geq 2\}$.
- Set of formulae:

$$\{\exists x_1 \dots x_n \bigwedge_{i \neq j \leq n} \neg x_i = x_j \mid n \in \mathbb{N}\}$$

  defines the class of infinite graphs.

A class of structures is called elementary, if there exists a set of $\mathcal{FO}$-formulae that defines the class.

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Frame definability

Which classes of Kripke frames are definable by a (set of) modal formulae.

Which elementary classes are definable by a (set of) modal formulae.

Examples:

Formula	Property of R	
$\Box p \to p$	Reflexive	$\forall w\,(wRw)$
$p \to \Box\Diamond p$	Symmetric	$\forall wv\,(wRv \to vRw)$
$\Box p \to \Box\Box p$	Transitive	$\forall wvu\,((wRv \wedge vRu) \to wRu)$
$\Diamond p \to \Box\Diamond p$	Euclidean	$\forall wvu\,((wRv \wedge wRu) \to vRu)$
$\Box p \to \Diamond p$	Serial	$\forall w \exists v\,(wRv)$

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Goldblatt-Thomason Theorem (1975)

Set Φ of atomic propositions. The formulae of $\mathcal{ML}(\Phi)$ are generated by:

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \vee \varphi) \mid \Box\varphi.$$

Theorem

An elementary frame class is $\mathcal{ML}$-definable iff

- *it is closed under taking*
  - *bounded morphic images*
  - *generated subframes*
  - *disjoint unions*
- *and its complement is closed under taking*
  - *ultrafilter extensions.*

Goldblatt-Thomason Theorem (Goranko, Passy 1992)

The formulae of $\mathcal{ML}(\boxed{u})$ are generated by:

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \vee \varphi) \mid \Box\varphi \mid \boxed{u}\,\varphi.$$

$$K, w \models \boxed{u}\,\varphi \quad \leftrightarrow \quad \forall v \in W : K, v \models \varphi.$$

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An elementary frame class is $\mathcal{ML}(\boxed{u})$-definable iff

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What do we study?

Frame definability of the fragment $\mathcal{ML}(\boxed{\mathrm{u}}^{+})$ of $\mathcal{ML}(\boxed{\mathrm{u}})$:

$$\varphi ::= p \mid \neg p \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid \Box\varphi \mid \Diamond\varphi \mid \boxed{\mathrm{u}}\, \varphi.$$

Frame definability of particular team based modal logics:

- Modal dependence logic $\mathcal{MDL}$.
- Extended modal dependence logic $\mathcal{EMDL}$.
- Modal logic with intuitionistic disjunction $\mathcal{ML}(\varovee)$.

What do we show?

- We give a variant of the Goldblatt-Thomason theorem for $\mathcal{ML}(\boxed{\mathsf{u}}^+)$.
- We show that with respect to frame definability:

$$\mathcal{ML} < \mathcal{MDL} = \mathcal{EMDL} = \mathcal{ML}(\varovee) = \mathcal{ML}(\boxed{\mathsf{u}}^+) < \mathcal{ML}(\boxed{\mathsf{u}}).$$

Frame definability in $\mathcal{ML}(\boxed{u}^+)$

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Every $\mathcal{ML}$-definable class is $\mathcal{ML}(\boxed{u}^+)$-definable, but not vice versa.

$\mathcal{ML}(\boxed{u}^+)$ is not closed under disjoint unions (e.g., $\boxed{u}\, p \vee \boxed{u} \neg p$).

Therefore $\mathcal{ML} <_F \mathcal{ML}(\boxed{u}^+)$.

Frame definability in $\mathcal{ML}(⊔^+)$

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Frame definability in $\mathcal{ML}(\Cup^+)$

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$\mathcal{ML}(\boxed{\text{u}}^+)$ is closed under generated subframes (e.g., $\Diamond_u \Diamond(p \vee \neg p)$).

Therefore $\mathcal{ML}(\boxed{\text{u}}^+) <_F \mathcal{ML}(\boxed{\text{u}})$.

Frame definability in $\mathcal{ML}(⩀^+)$

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Frame definability in $\mathcal{ML}(\boxed{u}^+)$

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Goldblatt-Thomason Theorem for $\mathcal{ML}(\boxed{u}^{+})$

Theorem (Does this suffice?)

An elementary frame class is $\mathcal{ML}(\boxed{u}^{+})$-definable iff

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NO! Something more is needed.

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Reflection of Finitely Generated Subframes

A frame class $\mathbb{F}$ reflects finitely generated subframes if:
whenever every finitely generated subframe of $\mathfrak{F}$ is in $\mathbb{F}$, then $\mathfrak{F}$ is also in $\mathbb{F}$.

Theorem

Every $\mathcal{ML}(\boxed{u}^+)$-definable frame class $\mathbb{F}$ reflects finitely generated subframes.

Goldblatt-Thomason theorem for $\mathcal{ML}(⊔^+)$

Theorem (Sano and V. 2015)

An elementary frame class $\mathbb{F}$ is $\mathcal{ML}(⊔^+)$-definable iff
$\mathbb{F}$ is closed under taking

- bounded morphic images & generated subframes

and it reflects

- ultrafilter extensions & finitely generated subframes.

$\because$ By van Benthem (1993)'s model theoretic argument.

Frame definability in team semantics

Def. $K \models \varphi$ iff $\forall T \subseteq W : K, T \models \varphi$ (iff $K, W \models \varphi$)

It is easy to show that $\mathcal{MDL} =_F \mathcal{EMDL}$.

Proof

Let φ be the dependence atom $\mathrm{dep}(\psi_1, \ldots, \psi_n)$, let k be the modal depth of φ, and let $p_1, \ldots, p_n$ be distinct fresh proposition symbols. Define

$$\varphi^* := \Big(\bigwedge_{0 \leq i \leq k} \Box^i \bigwedge_{1 \leq j \leq n} (p_j \leftrightarrow \psi_j)\Big) \rightarrow \mathrm{dep}(p_1, \ldots, p_n).$$

Next we will show that $\mathcal{ML}(\varovee) =_F \mathcal{ML}(\boxdot^+)$.

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Normal Forms for $\mathcal{ML}(\boxed{\text{u}}^{+})$ and $\mathcal{ML}(\varovee)$

Similar to the normal form for $\mathcal{ML}(\boxed{\text{u}})$ by Goranko and Passy 1992.

Proposition

With respect to frame definability $\mathcal{ML}(\boxed{\text{u}}^{+})$ and $\bigvee \boxed{\text{u}}\, \mathcal{ML}$ coincide.

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Normal Forms for $\mathcal{ML}(\boxed{u}^+)$ and $\mathcal{ML}(\varovee)$

Similar to the normal form for $\mathcal{ML}(\boxed{u})$ by Goranko and Passy 1992.

Proposition

With respect to frame definability $\mathcal{ML}(\boxed{u}^+)$ and $\bigvee \boxed{u}\, \mathcal{ML}$ coincide.

Proposition

Every $\mathcal{ML}(\varovee)$ formula is equivalent to a formula of the form $\bigovee_{i \leq n} \varphi_i$, where each φ_i is an $\mathcal{ML}$-formula.

Theorem

With respect to frame definability $\mathcal{ML}(\varovee)$ and $\bigvee \boxed{u}\, \mathcal{ML}$ coincide.

Normal Forms for $\mathcal{ML}(\boxdot^+)$ and $\mathcal{ML}(\ovee)$

Similar to the normal form for $\mathcal{ML}(\boxdot)$ by Goranko and Passy 1992.

Proposition

With respect to frame definability $\mathcal{ML}(\boxdot^+)$ and $\bigvee \boxdot \mathcal{ML}$ coincide.

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Every $\mathcal{ML}(\ovee)$ formula is equivalent to a formula of the form $\bigovee_{i \leq n} \varphi_i$, where each φ_i is an $\mathcal{ML}$-formula.

Theorem

With respect to frame definability $\mathcal{ML}(\ovee)$ and $\bigvee \boxdot \mathcal{ML}$ coincide.

Results

Theorem (Sano and V. 2015)

An elementary frame class $\mathbb{F}$ is $\mathcal{L}$-definable
($\mathcal{L} \in \{\mathcal{ML}(\varovee), \mathcal{MDL}, \mathcal{EMDL}, \mathcal{ML}(⊔^+)\}$) iff
$\mathbb{F}$ is closed under taking

- bounded morphic images & generated subframes

and it reflects

- ultrafilter extensions & finitely generated subframes.

Theorem (Sano and V. 2015)

With respect to frame definability:
$\mathcal{ML} < \mathcal{MDL} = \mathcal{EMDL} = \mathcal{ML}(\varovee) = \mathcal{ML}(⊔^+) < \mathcal{ML}(⊔)$.

Thanks!

Theorem (Sano and V. 2015)

An elementary frame class $\mathbb{F}$ is $\mathcal{L}$-definable
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Normal Form for $\mathcal{ML}(\boxdot^+)$

Similar to the normal form for $\mathcal{ML}(\boxdot)$ by Goranko and Passy 1992.

A formula φ is a closed disjunctive $\boxdot$-clause if

φ is of the form $\bigvee_{i \in I} \boxdot \psi_i$ ($\psi_i \in \mathcal{ML}$).

A formula φ is in conjunctive $\boxdot$-form if

φ is of the form $\bigwedge_{j \in J} \psi_j$, where each ψ_j is a closed disjunctive $\boxdot$-clause.

Theorem

Each formula of $\mathcal{ML}(\boxdot^+)$ is equivalent to a formula in conjunctive $\boxdot$-form.

Corollary

With respect to frame definability $\mathcal{ML}(\boxdot^+)$ and $\bigvee \boxdot \mathcal{ML}$ coincide.

Normal Form for $\mathcal{ML}(\boxed{u}^+)$

Similar to the normal form for $\mathcal{ML}(\boxed{u})$ by Goranko and Passy 1992.

A formula φ is a closed disjunctive $\boxed{u}$-clause if

φ is of the form $\bigvee_{i \in I} \boxed{u}\, \psi_i$ $(\psi_i \in \mathcal{ML})$.

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Each formula of $\mathcal{ML}(\boxed{u}^+)$ is equivalent to a formula in conjunctive $\boxed{u}$-form.

Corollary

With respect to frame definability $\mathcal{ML}(\boxed{u}^+)$ and $\bigvee \boxed{u}\, \mathcal{ML}$ coincide.

Normal Form for $\mathcal{ML}(\boxed{\text{u}}^{+})$

Similar to the normal form for $\mathcal{ML}(\boxed{\text{u}})$ by Goranko and Passy 1992.

A formula φ is a closed disjunctive $\boxed{\text{u}}$-clause if
φ is of the form $\bigvee_{i \in I} \boxed{\text{u}}\, \psi_i$ ($\psi_i \in \mathcal{ML}$).

A formula φ is in conjunctive $\boxed{\text{u}}$-form if
φ is of the form $\bigwedge_{j \in J} \psi_j$, where each ψ_j is a closed disjunctive $\boxed{\text{u}}$-clause.

Theorem

Each formula of $\mathcal{ML}(\boxed{\text{u}}^{+})$ is equivalent to a formula in conjunctive $\boxed{\text{u}}$-form.

Corollary

With respect to frame definability $\mathcal{ML}(\boxed{\text{u}}^{+})$ and $\bigvee \boxed{\text{u}}\, \mathcal{ML}$ coincide.

Normal Form for $\mathcal{ML}(\boxed{u}^+)$

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With respect to frame definability $\mathcal{ML}(\boxed{u}^+)$ and $\bigvee \boxed{u}\, \mathcal{ML}$ coincide.

Normal Form for $\mathcal{ML}(\varovee)$

Every formula is equivalent to a formula of the form

$$\bigovee_{i \leq n} \varphi_i,$$

where each φ_i is an $\mathcal{ML}$-formula.

Theorem

With respect to frame definability $\mathcal{ML}(\varovee)$ and $\bigvee \boxdot \mathcal{ML}$ coincide.

(Already in the level of validity in a model.)

Normal Form for $\mathcal{ML}(\varovee)$

Every formula is equivalent to a formula of the form

$$\bigvarovee_{i \leq n} \varphi_i,$$

where each φ_i is an $\mathcal{ML}$-formula.

Theorem

With respect to frame definability $\mathcal{ML}(\varovee)$ and $\bigvee \boxdot \mathcal{ML}$ coincide.

(Already in the level of validity in a model.)

Normal Form for $\mathcal{ML}(\varovee)$

Every formula is equivalent to a formula of the form

$$\bigovee_{i \leq n} \varphi_i,$$

where each φ_i is an $\mathcal{ML}$-formula.

Theorem

With respect to frame definability $\mathcal{ML}(\varovee)$ and $\bigvee \boxed{u}\, \mathcal{ML}$ coincide.

(Already in the level of validity in a model.)

Bounded morphism and Ultrafilter Extension

$f : (W, R) \to (W', R')$ is a bounded morphism if:

- (Forth) wRv implies $f(w)R'f(v)$
- (Back) $f(w)R'b$ implies: $f(v) = b$ and wRv for some v

$(\mathrm{Uf}(W), R^{\mathrm{ue}})$ is the ultrafilter extension of (W, R) where:

- $\mathrm{Uf}(W)$ is the set of all ultrafilters $\mathcal{U} \subseteq \mathcal{P}(W)$.
- $\mathcal{U} R^{\mathrm{ue}} \mathcal{U}'$ iff $Y \in \mathcal{U}'$ implies $R^{-1}[Y] \in \mathcal{U}$ for all $Y \subseteq W$.