Temporal Team Semantics Revisited

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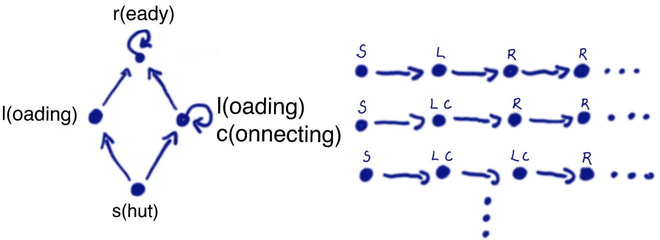
Traceproperties and hyperproperties

Opening your office computer after holidays:



Traceproperties hold in a system if each trace (in isolation) has the property:

- The computer will be eventually ready.

Hyperproperties are properties of sets of traces (in security, information flow, etc.):

- The computer will be ready in bounded time.
- Public outputs do not leak information about secret outputs.

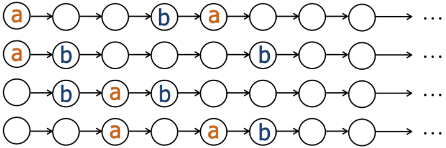
Two paradigms: HyperLTL vs. TeamLTL

A trace-set T satisfies $\varphi \vee \psi$ if it decomposes to sets T_φ and T_ψ satisfying φ and ψ.

$$(T, i) \models \varphi \vee \psi \text{ iff } (T_1, i) \models \varphi \text{ and } (T_2, i) \models \psi, \text{ for some } T_1 \cup T_2 = T$$

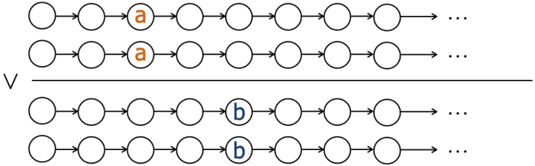
HyperLTL:

$\forall \pi . \forall \pi' .\ \mathrm{F}((a_\pi \wedge a_{\pi'}) \vee (b_\pi \wedge b_{\pi'}))$



TeamLTL:

$(\mathrm{F}\ a) \vee (\mathrm{F}\ b)$



LTL, HyperLTL, and TeamLTL

In LTL the satisfying object is a trace: $T \models \varphi$ iff $\forall t \in T : t \models \varphi$

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \vee \varphi) \mid X\varphi \mid \varphi U \varphi$$

In HyperLTL the satisfying object is a set of traces and a trace assignment: $\Pi \models_T \varphi$

$$\varphi ::= \exists\pi\varphi \mid \forall\pi\varphi \mid \psi$$
$$\psi ::= p_\pi \mid \neg\psi \mid (\psi \vee \psi) \mid X\psi \mid \psi U \psi$$

In TeamLTL the satisfying object is a set of traces. We use team semantics: $(T, i) \models \varphi$

$$\varphi ::= p \mid \neg p \mid (\varphi \vee \varphi) \mid (\varphi \wedge \varphi) \mid X\varphi \mid \varphi U \mid \varphi W \varphi$$

+ new atomic statements (dependence and inclusion atoms: $\mathrm{dep}(\vec{p}, q)$, $\vec{p} \subseteq \vec{q}$)
+ additional connectives (Boolean disjunction, contradictory negation, etc.)

Extensions are a well-defined way to delineate expressivity and complexity

TeamLTL with asynchronous behaviour

- Synchronous TeamLTL (Krebs, Meier, V., and Zimmermann, MFCS 2018):
  - Collection of traces T with one global clock i.
  - $(T, i) \models \mathsf{F}\varphi \quad \text{iff} \quad (T, i+k) \models \varphi$, for some $k \in \mathbb{N}$
- Asynchronous TeamLTL (Krebs, Meier, V., and Zimmermann, MFCS 2018):
  - Collection of traces T with a collection of local clocks $f\colon T \to \mathbb{N}$.
  - Local clocks are completely independent.
  - $(T, f) \models \mathsf{F}\varphi \quad \text{iff} \quad (T, g) \models \varphi$, for some g s.t. $g(t) \geq f(t)$, for each $t \in T$.

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- TeamLTL with time evaluation functions (tefs) (Gutsfeld, Meier, Ohrem, and V., LICS 2022):
  - Collection of traces T and a tef $\tau\colon \mathbb{N} \times T \to \mathbb{N}$ relating a global clock to local clocks.
  - The behaviour of local clocks is determined by a tef.
  - $(T, \tau) \models \mathsf{F}\varphi \quad$ iff $\quad (T, \tau[k, \infty]) \models \varphi$, for some $k \in \mathbb{N}$.
  - Synchronous TeamLTL is an instance, where the tef is synchronous!
  - (cf. trajectories of Bonakdarpour, Prabhakar, Sánchez, NASA Formal Methods 2020)

Properties of tefs

Property	**Definition**
Monotonicity	$\forall i \in \mathbb{N} : \tau(i) \leq \tau(i+1)$
Strict Monotonicity	$\forall i \in \mathbb{N} : \tau(i) < \tau(i+1)$
Stepwiseness	$\forall i \in \mathbb{N} : \tau(i) \leq \tau(i+1) \leq \tau(i) + \vec{1}$
*Fairness	$\forall i \in \mathbb{N}\, \forall t \in T\, \exists j \in \mathbb{N} : \tau(j,t) \geq i$
*Non-Parallelism	$\forall i \in \mathbb{N} : i = \sum_{t \in T} \tau(i,t)$
*Synchronicity	$\forall i, i' \in \mathbb{N}\, \forall t \in T : \tau(i,t) = \tau(i,t')$

Table: * are optional. $\tau(i)$ is the tuple $\big(\tau(i,t)\big)_{t \in T}$ of values of local clocks at time i.

- stuttering tef satisfies monotonicity
- tef satisfies strict monotonicity and stepwiseness
- synchronous tef satisfies strict monotonicity, stepwiseness, and synchronicity

Team semantics with tefs

Let (T, τ) be a pair, where T is a multiset of traces and τ is a stuttering tef for T.

$$
\begin{array}{lll}
(T, \tau) \models \mathrm{X}\varphi & \text{iff} & (T, \tau[1, \infty]) \models \varphi \\
(T, \tau) \models [\varphi \mathrm{U} \psi] & \text{iff} & \exists k \in \mathbb{N} \text{ such that } (T, \tau[k, \infty]) \models \psi \text{ and} \\
& & \quad \forall m : 0 \leq m < k \Rightarrow (T, \tau[m, \infty]) \models \varphi
\end{array}
$$

k-shifted tef if defined by $\tau[k, \infty](i, t) := \tau(i + k, t)$, for all $t \in T$, $i \in \mathbb{N}$.

Variants and extensions of TeamLTL

$\exists$TeamLTL

- $T \models_{\exists} \varphi$ if $(T, \tau) \models \varphi$ for some initial tef of T.

$\forall$TeamLTL

- $T \models_{\forall} \varphi$ if $(T, \tau) \models \varphi$ for all initial tefs of T.

Synchronous TeamLTL

- $T \models_{s} \varphi$ if $(T, \tau) \models \varphi$ for the unique initial synchronous tef of T.

$\mathrm{TeamCTL}^*$ has the same syntax as CTL^*:

$$\varphi ::= p \mid \neg p \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \mathrm{X}\varphi \mid \varphi \mathrm{U} \varphi \mid \varphi \mathrm{W} \varphi \mid \exists \varphi \mid \forall \varphi$$

The quantifiers $\exists$ and $\forall$ range over tefs:

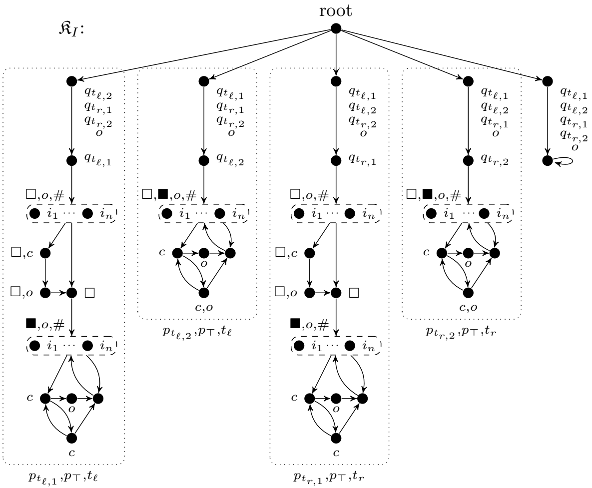
$$(T, \tau) \models \exists \varphi \text{ iff } (T, \tau') \models \varphi \text{ for some tef } \tau' \text{ of } T \text{ s.t. } \tau'(0) = \tau(0),$$
$$(T, \tau) \models \forall \varphi \text{ iff } (T, \tau') \models \varphi \text{ for all tefs } \tau' \text{ of } T \text{ s.t. } \tau'(0) = \tau(0).$$

Complexity results: When are the logics decidable?

Model Checking Problem for	Complexity
$\exists\mathrm{TeamLTL}(\otimes, \mathrm{NE})$	Σ^0_1-hard
$\exists\mathrm{TeamCTL}^*(\otimes)$	Σ^1_1-hard
$\mathrm{TeamCTL}^*(\mathcal{S}, \mathrm{ALL})$ for k-synchronous or k-context-bounded tefs, when the team is finite	decidable
Fixed formula, k, and team size	polynomial time

Table: ALL is the set of all "generalised atoms" and $\mathcal{S} = \{\otimes, \mathrm{NE}, \overset{1}{\mathrm{A}}, \mathrm{dep}, \subseteq\}$.
Decidability via a translation to Alternating Asynchronous Büchi Automata, (Gutsfeld, Müller-Olm, and Ohrem, POPL 2021).

Gadgets for recurrent 2-counter machines



Advertisements

- Workshop 30 years of finite model theory in Finland, Helsinki, Finland.
  - Date: August 21-23 2022
  - Registration for giving a talk and to participate: July 6th 2022
  - Website: `https://www.helsinki.fi/en/conferences/30-years-of-finite-model-theory-in-finland`
  - Contact: `fmt-30@helsinki.fi`
- 2 x PostDoc positions in Computer Science Logic, University of Sheffield, UK
  - Topic: Logics and complexity theory utilising numerical features and real valued data
  - Duration: until 31.10.2023 (2 x 15m, might be possible to join to one longer post)
  - Deadline: July 20th 2022.
- PhD position in Temporal Logic, University of Sheffield, UK
  - Fully funded for 3.5 years for students applicable for UK Home rates (Brexit)
  - International applicants: Graduate teaching assistant, etc. possibilities in near future.
- Details: `www.virtema.fi`
- Contact: `j.t.virtema@sheffield.ac.uk`