

Linear-time Temporal Logic with Team Semantics: Expressivity and Complexity

Jonni Virtema¹ Jana Hofmann²
Bernd Finkbeiner² Juha Kontinen³ Fan Yang³

¹ University of Sheffield, UK

² CISPA Helmholtz Center for Information Security, Germany

³ University of Helsinki Finland

15.12.2021 — FSTTCS'21

Logics for verification and specification of concurrent systems

Basic setting:

- ▶ **System** (e.g., piece of software or hardware)
 \rightsquigarrow **Kripke structure** depicting the behaviour of the system
- ▶ A single **run** of the system
 \rightsquigarrow a **trace** generated by the Kripke structure
- ▶ A **property** of the system (e.g., every request is eventually granted)
 \rightsquigarrow a **formula** of some formal language expressing the property.

Logics for verification and specification of concurrent systems

Basic setting:

- ▶ **System** (e.g., piece of software or hardware)
 \rightsquigarrow **Kripke structure** depicting the behaviour of the system
- ▶ A single **run** of the system
 \rightsquigarrow a **trace** generated by the Kripke structure
- ▶ A **property** of the system (e.g., every request is eventually granted)
 \rightsquigarrow a **formula** of some formal language expressing the property.

Model checking:

- ▶ Check whether a given **system satisfies** a given **specification**.

SAT solving:

- ▶ Check whether a given **specification** (or collection of) **can be realised**.

Snapshot of our paper

State of the art:

- ▶ LTL, QPTL, CTL, etc. vs. HyperLTL, HyperQPTL, HyperCTL, etc.
are prominent logics for **traceproperties** vs. **hyperproperties** of systems
 - ▶ Traceproperty: Each request is eventually granted (**properties of traces**)
 - ▶ Hyperproperty: Each request is granted in bounded time (properties of **sets of traces**)
- ▶ HyperLogics are of **high complexity** or undecidable.
Not well suited for properties involving **unbounded number** of traces.

Snapshot of our paper

State of the art:

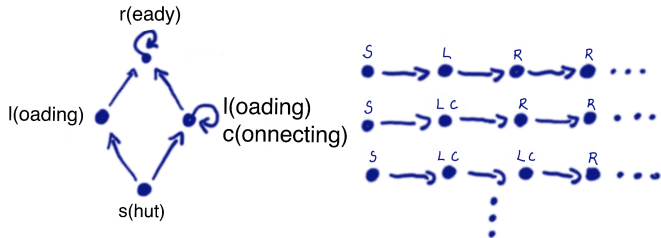
- ▶ LTL, QPTL, CTL, etc. vs. HyperLTL, HyperQPTL, HyperCTL, etc.
are prominent logics for **traceproperties** vs. **hyperproperties** of systems
 - ▶ Traceproperty: Each request is eventually granted (**properties of traces**)
 - ▶ Hyperproperty: Each request is granted in bounded time (properties of **sets of traces**)
- ▶ HyperLogics are of **high complexity** or undecidable.
Not well suited for properties involving **unbounded number** of traces.

This paper:

- ▶ Temporal logics with **team semantics** for expressing hyperproperties
Purely modal logic & well suited for properties of **unbounded number** of traces.
- ▶ Expressivity: We relate variants of TeamLTL to HyperLogics
- ▶ Complexity: We explore the undecidability frontier of TeamLTL extensions
 - ▶ Discovered a large EXPTIME fragment: **left-flat and downward closed** logics
 - ▶ Already TeamLTL with **inclusion atoms and Boolean disjunctions** is undecidable

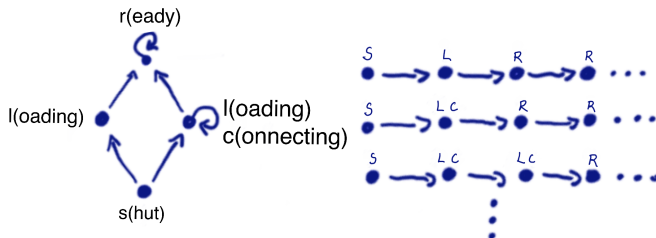
Traceproperties and hyperproperties

Opening your office computer after holidays:



Traceproperties and hyperproperties

Opening your office computer after holidays:



Traceproperties hold in a system if **each trace** (in isolation) **has the property**:

- ▶ The computer will be **eventually ready** (or will be loading forever).

Hyperproperties are **properties of sets of traces**:

- ▶ The computer will be **ready in bounded time**.

Logics for **traceproperties** and hyperproperties

- ▶ **Linear-time temporal logic** (LTL) is one of the most **prominent logics** for the **specification and verification** of reactive and concurrent systems.
- ▶ Model checking **tools** like SPIN and NuSMV **automatically verify** whether a given computer system is correct with respect to its **LTL specification**.
- ▶ One reason for the success of LTL over first-order logic is that LTL is a **purely modal logic** and thus has many desirable properties.
 - ▶ LTL is decidable (**PSPACE-complete** model checking and satisfiability).
 - ▶ $\text{FO}^2(\leq)$ and $\text{FO}^3(\leq)$ SAT are **NEXPTIME-complete and non-elementary**.
 - ▶ LTL is **bisimulation invariant** (cannot separate systems whose traces behave similarly)
- ▶ Caveat: LTL can specify **only traceproperties**.

Logics for traceproperties and hyperproperties

Recipe for logics for hyperproperties:

A logic for traceproperties \rightsquigarrow add trace quantifiers

In LTL the satisfying object is a trace: $T \models \varphi$ iff $\forall t \in T : t \models \varphi$

$$\varphi ::= p \mid \neg \varphi \mid (\varphi \vee \varphi) \mid X\varphi \mid \varphi U \varphi$$

In HyperLTL the satisfying object is a set of traces and a trace assignment: $\Pi \models_T \varphi$

$$\varphi ::= \exists \pi \varphi \mid \forall \pi \varphi \mid \psi$$

$$\psi ::= p_\pi \mid \neg \psi \mid (\psi \vee \psi) \mid X\psi \mid \psi U \psi$$

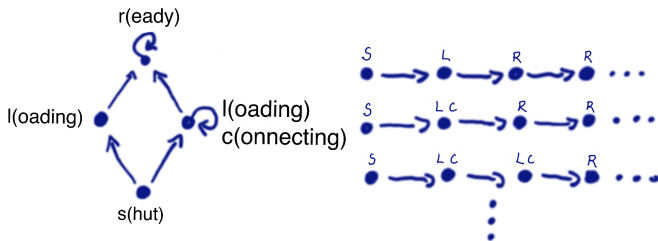
HyperQPTL extends HyperLTL by (uniform) quantification of propositions: $\exists p \varphi, \forall p \varphi$

Logics for traceproperties and hyperproperties

- ▶ Quantification based logics for hyperproperties: HyperLTL, HyperCTL, etc.
- ▶ Retain some desirable properties of LTL, but are not purely modal logics
 - ▶ Model checking for \exists^* HyperLTL and HyperLTL are PSPACE and non-elementary.
 - ▶ HyperLTL satisfiability is highly undecidable.
 - ▶ HyperLTL formulae express properties expressible using fixed finite number of traces.

Logics for traceproperties and hyperproperties

- ▶ Quantification based logics for hyperproperties: HyperLTL, HyperCTL, etc.
- ▶ Retain some desirable properties of LTL, but are **not purely modal logics**
 - ▶ Model checking for \exists^* HyperLTL and HyperLTL are PSPACE and non-elementary.
 - ▶ HyperLTL satisfiability is highly undecidable.
 - ▶ HyperLTL formulae express properties expressible using fixed finite number of traces.
- ▶ Bounded termination is not definable in HyperLTL (but is in HyperQPTL)



- ▶ Team semantics is a candidate for a purely modal logic without the above caveat.

Core of Team Semantics

- ▶ In most studied logics formulae are evaluated in a single state of affairs.

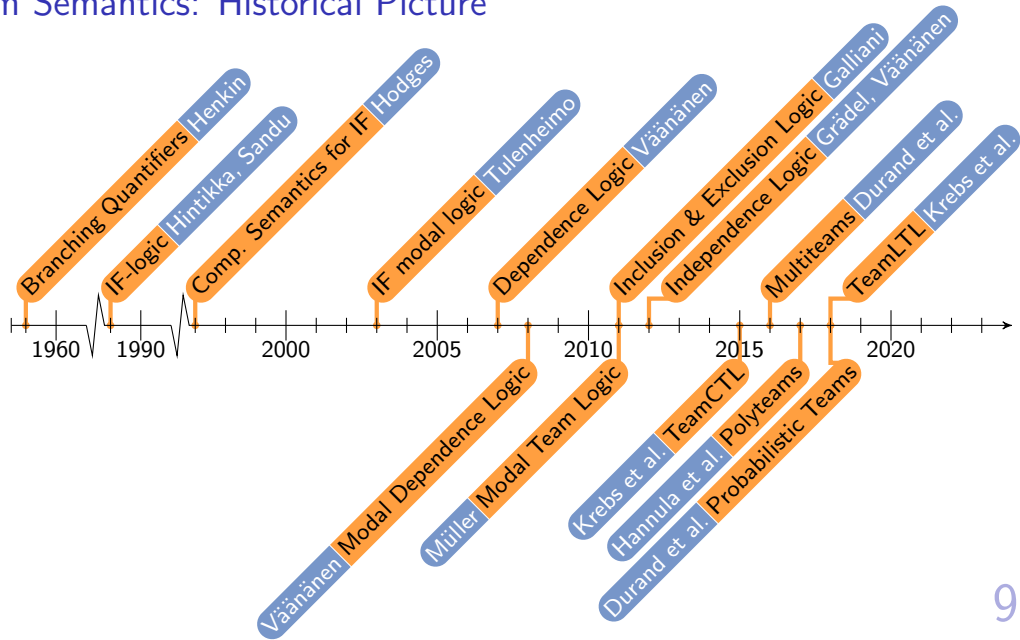
E.g.,

- ▶ a first-order assignment in first-order logic,
- ▶ a propositional assignment in propositional logic,
- ▶ a possible world of a Kripke structure in modal logic.

Core of Team Semantics

- ▶ In most studied logics formulae are evaluated in a single state of affairs.
E.g.,
 - ▶ a first-order assignment in first-order logic,
 - ▶ a propositional assignment in propositional logic,
 - ▶ a possible world of a Kripke structure in modal logic.
- ▶ In **team** semantics **sets** of states of affairs are considered.
E.g.,
 - ▶ a **set** of first-order assignments in first-order logic,
 - ▶ a **set** of propositional assignments in propositional logic,
 - ▶ a **set** of possible worlds of a Kripke structure in modal logic.
- ▶ These sets of things are called **teams**.

Team Semantics: Historical Picture



LTL, HyperLTL, and TeamLTL

In LTL the satisfying object is a **trace**: $T \models \varphi$ iff $\forall t \in T : t \models \varphi$

$$\varphi ::= p \mid \neg \varphi \mid (\varphi \vee \varphi) \mid X\varphi \mid \varphi U \varphi$$

In HyperLTL the satisfying object is a **set of traces** and a **trace assignment**: $\Pi \models_T \varphi$

$$\varphi ::= \exists \pi \varphi \mid \forall \pi \varphi \mid \psi$$

$$\psi ::= p_\pi \mid \neg \psi \mid (\psi \vee \psi) \mid X\psi \mid \psi U \psi$$

In TeamLTL the satisfying object is a **set of traces**. We use **team semantics**: $(T, i) \models \varphi$

$$\varphi ::= p \mid \neg p \mid (\varphi \vee \varphi) \mid (\varphi \wedge \varphi) \mid X\varphi \mid \varphi U \mid \varphi W \varphi$$

+ new atomic statements (**dependence** and **inclusion** atoms: $\text{dep}(\vec{p}, q)$, $\vec{p} \subseteq \vec{q}$)

+ additional connectives (Boolean disjunction, contradictory negation, etc.)

Extensions are a well-defined **way to delineate expressivity and complexity**

Examples: HyperLTL vs. TeamLTL

Temporal team semantics is **universal** and **synchronous**

$$(T, i) \models p \text{ iff } \forall t \in T : t[i](p) = 1 \quad (T, i) \models \neg p \text{ iff } \forall t \in T : t[i](p) = 0$$

$$(T, i) \models F\varphi \text{ iff } (T, j) \models \varphi \text{ for some } j \geq i \quad (T, i) \models G\varphi \text{ iff } (T, j) \models \varphi \text{ for all } j \geq i$$

Examples: HyperLTL vs. TeamLTL

Temporal team semantics is **universal** and **synchronous**

$$(T, i) \models p \text{ iff } \forall t \in T : t[i](p) = 1 \quad (T, i) \models \neg p \text{ iff } \forall t \in T : t[i](p) = 0$$

$$(T, i) \models F\varphi \text{ iff } (T, j) \models \varphi \text{ for some } j \geq i \quad (T, i) \models G\varphi \text{ iff } (T, j) \models \varphi \text{ for all } j \geq i$$

There is a timepoint (common for all traces) after which **a** does not occur.

Not expressible in HyperLTL, but is in **HyperQPTL**.

$$\exists p \forall \pi Fp \wedge G(p \rightarrow G\neg a_\pi)$$

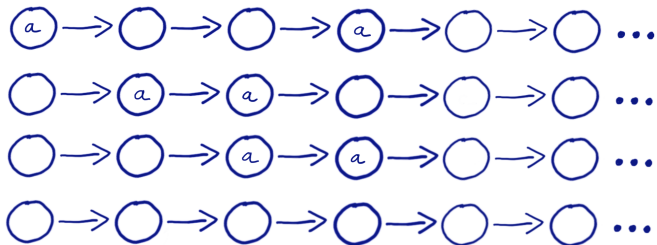
Expressible in synchronous TeamLTL: **FG $\neg a$**

Examples: HyperLTL vs. TeamLTL

There is a timepoint (common for all traces) after which a does not occur.
Not expressible in HyperLTL, but is in **HyperQPTL**.

$$\exists p \forall \pi Fp \wedge G(p \rightarrow G\neg a_\pi)$$

Expressible in synchronous TeamLTL: $FG \neg a$

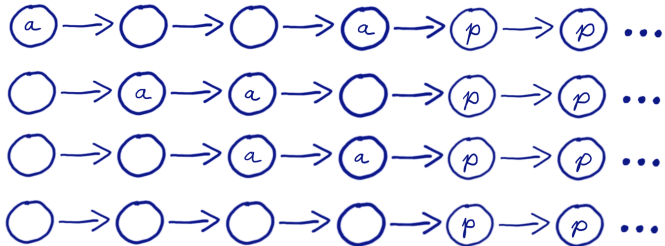


Examples: HyperLTL vs. TeamLTL

There is a timepoint (common for all traces) after which a does not occur.
Not expressible in HyperLTL, but is in **HyperQPTL**.

$$\exists p \forall \pi Fp \wedge G(p \rightarrow G\neg a_\pi)$$

Expressible in synchronous TeamLTL: $FG\neg a$

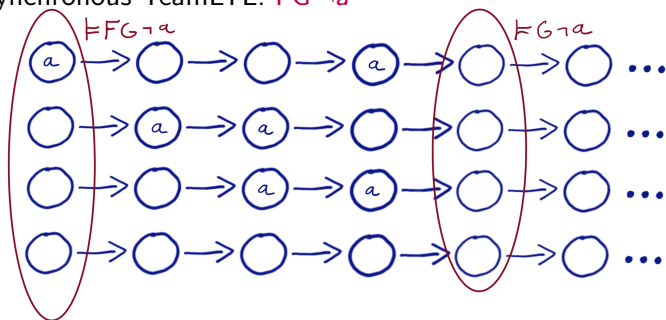


Examples: HyperLTL vs. TeamLTL

There is a timepoint (common for all traces) after which a does not occur.
Not expressible in HyperLTL, but is in **HyperQPTL**.

$$\exists p \forall \pi Fp \wedge G(p \rightarrow G\neg a_\pi)$$

Expressible in synchronous TeamLTL: $FG\neg a$



Examples: HyperLTL vs. TeamLTL

A **trace-set** T satisfies $\varphi \vee \psi$ if it **decomposed** to sets T_φ and T_ψ satisfying φ and ψ .

$(T, i) \models \varphi \vee \psi$ iff $(T_1, i) \models \varphi$ and $(T_2, i) \models \psi$, for some $T_1 \cup T_2 = T$

$(T, i) \models \varphi \wedge \psi$ iff $(T, i) \models \varphi$ and $(T, i) \models \psi$

Examples: HyperLTL vs. TeamLTL

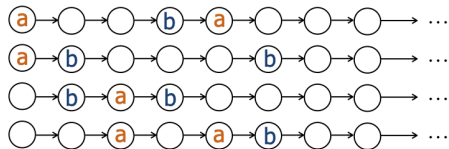
A **trace-set** T satisfies $\varphi \vee \psi$ if it **decomposed** to sets T_φ and T_ψ satisfying φ and ψ .

$(T, i) \models \varphi \vee \psi$ iff $(T_1, i) \models \varphi$ and $(T_2, i) \models \psi$, for some $T_1 \cup T_2 = T$

$(T, i) \models \varphi \wedge \psi$ iff $(T, i) \models \varphi$ and $(T, i) \models \psi$

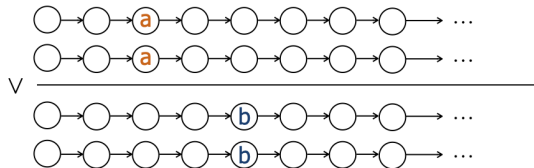
HyperLTL:

$\forall \pi. \forall \pi'. F((a_\pi \wedge a_{\pi'}) \vee (b_\pi \wedge b_{\pi'}))$



TeamLTL:

$(F a) \vee (F b)$



Examples: HyperLTL vs. TeamLTL

Dependence atom $\text{dep}(p_1, \dots, p_m, q)$ states that p_1, \dots, p_m functionally determine q :

$$(T, i) \models \text{dep}(p_1, \dots, p_m, q) \text{ iff } \forall t, t' \in T \left(\bigwedge_{1 \leq j \leq m} t[i](p_j) = t'[i](p_j) \right) \Rightarrow (t[i](q) = t'[i](q))$$

Examples: HyperLTL vs. TeamLTL

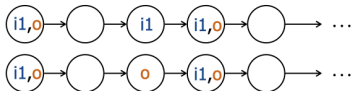
Dependence atom $\text{dep}(p_1, \dots, p_m, q)$ states that p_1, \dots, p_m functionally determine q :

$$(T, i) \models \text{dep}(p_1, \dots, p_m, q) \text{ iff } \forall t, t' \in T \left(\bigwedge_{j=1}^m t[i](p_j) = t'[i](p_j) \right) \Rightarrow (t[i](q) = t'[i](q))$$

TeamLTL:

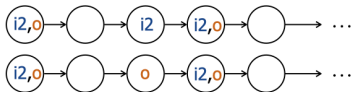
$$(G \text{ dep}(i1, o)) \vee (G \text{ dep}(i2, o))$$

Nondeterministic dependence: “ o either depends on $i1$ or on $i2$ ”



“whenever the traces agree on $i1$, they agree on o ”

\vee



“whenever the traces agree on $i2$, they agree on o ”

Examples: HyperLTL vs. TeamLTL

Boolean disjunction: $(T, i) \models \varphi \oplus \psi$ iff $(T, i) \models \varphi$ or $(T, i) \models \psi$.

Depending on an unknown input, execution traces either agree on a or on b .

Expressible in HyperLTL with three trace quantifiers:

$$\exists \pi_1 \exists \pi_2 \forall \pi G(a_{\pi_1} \leftrightarrow a_{\pi}) \vee G(b_{\pi_2} \leftrightarrow b_{\pi}).$$

Expressible in TeamLTL:

$$G \text{ dep}(a) \vee G \text{ dep}(b) \text{ and } G(a \oplus \neg a) \vee G(b \oplus \neg b).$$

Quantification of traces in TeamLTL

Inclusion atom $p_1 \dots p_n \subseteq q_1 \dots q_n$ states: truth-values of $p_1 \dots p_n$ occur for $q_1 \dots q_n$.

$$(T, i) \models p_1 \dots p_n \subseteq q_1 \dots q_n \text{ iff } \forall t \exists t' t[i](p_1) = t'[i](q_1), \dots, t[i](p_n) = t'[i](q_n)$$

Inclusion atoms can be used to express traceproperties in TeamLTL:

- ▶ $\forall \pi. \varphi_\pi$ can be expressed with $\varphi \subseteq \top$.
- ▶ $\exists \pi. \varphi_\pi$ can be expressed with $\top \subseteq \varphi$.

Quantification of traces in TeamLTL

Inclusion atom $p_1 \dots p_n \subseteq q_1 \dots q_n$ states: truth-values of $p_1 \dots p_n$ occur for $q_1 \dots q_n$.

$$(T, i) \models p_1 \dots p_n \subseteq q_1 \dots q_n \text{ iff } \forall t \exists t' t[i](p_1) = t'[i](q_1), \dots, t[i](p_n) = t'[i](q_n)$$

Inclusion atoms can be used to express traceproperties in TeamLTL:

- ▶ $\forall \pi. \varphi_\pi$ can be expressed with $\varphi \subseteq \top$.
- ▶ $\exists \pi. \varphi_\pi$ can be expressed with $\top \subseteq \varphi$.

Some properties involving single quantifier blocks can be expressed in TeamLTL.

- ▶ $\Pi \models_T \forall \pi_1 \dots \forall \pi_n. \varphi_{\vec{\pi}}$ is related to $(T', 0) \models \varphi$ for all subteams $T' \subseteq T$ of size at most n .
- ▶ $\Pi \models_T \exists \pi_1 \dots \exists \pi_n. \varphi_{\vec{\pi}}$ is related to $(T', 0) \models \varphi$ for some subteam $T' \subseteq T$ of size at most n .

No obvious way to mimic quantifier alternation without encoding gadgets to traces.

Temporal team semantics

Definition

Temporal team is (T, i) , where T a set of traces and $i \in \mathbb{N}$.

$(T, i) \models p$	iff	$\forall t \in T : t[0](p) = 1$
$(T, i) \models \neg p$	iff	$\forall t \in T : t[0](p) = 0$
$(T, i) \models \phi \wedge \psi$	iff	$(T, i) \models \phi$ and $(T, i) \models \psi$
$(T, i) \models \phi \vee \psi$	iff	$(T_1, i) \models \phi$ and $(T_2, i) \models \psi$, for some T_1, T_2 s.t. $T_1 \cup T_2 = T$
$(T, i) \models X\varphi$	iff	$(T, i+1) \models \varphi$
$(T, i) \models \phi U \psi$	iff	$\exists k \geq i$ s.t. $(T, k) \models \psi$ and $\forall m : i \leq m < k \Rightarrow (T, m) \models \phi$
$(T, i) \models \phi W \psi$	iff	$\forall k \geq i : (T, k) \models \phi$ or $\exists m$ s.t. $i \leq m \leq k$ and $(T, m) \models \psi$

As usual $F\varphi := (\top U \varphi)$ and $G\varphi := (\varphi W \perp)$.

TeamLTL(\otimes, \subseteq) is the extension with the atoms and extra connectives in the brackets.

Motivation of the current work

- ▶ recent interest into **temporal** team semantics
[Krebs et al 2018, Lück 2020, Kontinen & Sandsrtöm 2021, Gutsfeld et al. 2021]
- ▶ develop purely modal logics for **hyperproperties**
- ▶ discover decidable and expressive logics for hyperproperties
- ▶ investigate connections between HyperLTL and TeamLTL variants

Results of our paper

Generalised atoms and complete logics

Let B be a set of n -ary Boolean relations. We define the property $[\varphi_1, \dots, \varphi_n]_B$ for an n -tuple $(\varphi_1, \dots, \varphi_n)$ of LTL-formulae:

$$(T, i) \models [\varphi_1, \dots, \varphi_n]_B \quad \text{iff} \quad \{(\llbracket \phi_1 \rrbracket_{(t,i)}, \dots, \llbracket \phi_n \rrbracket_{(t,i)}) \mid t \in T\} \in B.$$

Generalised atoms and complete logics

Let B be a set of n -ary Boolean relations. We define the property $[\varphi_1, \dots, \varphi_n]_B$ for an n -tuple $(\varphi_1, \dots, \varphi_n)$ of LTL-formulae:

$$(T, i) \models [\varphi_1, \dots, \varphi_n]_B \quad \text{iff} \quad \{(\llbracket \phi_1 \rrbracket_{(t,i)}, \dots, \llbracket \phi_n \rrbracket_{(t,i)}) \mid t \in T\} \in B.$$

Theorem

$\text{TeamLTL}(\emptyset, \text{NE}, \overset{1}{A})$ can express all $[\varphi_1, \dots, \varphi_n]_B$.

$\text{TeamLTL}(\emptyset, \overset{1}{A})$ can express all $[\varphi_1, \dots, \varphi_n]_B$, for *downward closed* B .

- ▶ B is downward closed if $S_1 \in B$ & $S_2 \subseteq S_1$ imply $S_2 \in B$.
- ▶ $(T, i) \models \text{NE}$ iff $T \neq \emptyset$.
- ▶ $(T, i) \models A\varphi$ iff $(T', i) \models \varphi$, for all $T' \subseteq T$.
- ▶ $(T, i) \models \overset{1}{A}\varphi$ iff $(\{t\}, i) \models \varphi$, for all $t \in T$.

Complexity results

Logic	Model Checking Result
TeamLTL without \vee	in PSPACE [Krebs et al. 2018]
k -coherent TeamLTL(\sim)	in EXPSPACE
left-flat TeamLTL($\oplus, \overset{1}{A}$)	in EXPSPACE
TeamLTL(\subseteq, \oplus)	Σ_1^0 -hard
TeamLTL(\subseteq, \oplus, A)	Σ_1^1 -hard
TeamLTL(\sim)	complete for third-order arithmetic [Luck 2020]

Table: Complexity results.

- ▶ k -coherence: $(T, i) \models \varphi$ iff $(S, i) \models \varphi$ for all $S \subseteq T$ s.t. $|S| \leq k$
- ▶ left-flatness: Restrict U and W syntactically to $(\overset{1}{A}\varphi U\psi)$ and $(\overset{1}{A}\varphi W\psi)$
- ▶ \sim is contradictory negation and TeamLTL(\sim) subsumes all the other logics

Source of inclusion results

$$\begin{array}{ll}
 \text{TeamLTL}(\oplus, \overset{1}{\mathbf{A}}) & \leq \quad \overset{u}{\exists}_q^* \forall_\pi \text{HyperQPTL} \text{ (assuming left-flatness)} \\
 & \leq \quad \exists_p \overset{u}{Q}_p^* \forall_\pi \text{HyperQPTL}^+ \text{ (general case)} \\
 \wedge^\dagger & \\
 \text{TeamLTL}(\oplus, \text{NE}, \overset{1}{\mathbf{A}}) & \leq \quad \exists_p \overset{u}{Q}_p^* \exists_\pi^* \forall_\pi \text{HyperQPTL}^+ \\
 |\wedge \quad [\text{Luck 2020}] & \text{(assuming } k\text{-coherence)} \\
 \text{TeamLTL}(\sim) & \leq \quad \forall^k \text{HyperLTL}
 \end{array}$$

Table: Expressivity results. \dagger holds since $\text{TeamLTL}(\overset{1}{\mathbf{A}}, \oplus)$ is downward closed.

Source of Undecidability

Definition

A **non-deterministic 3-counter machine** M consists of a list I of n instructions that manipulate three counters C_l , C_m and C_r . All instructions are of the following forms:

- ▶ C_a^+ goto $\{j_1, j_2\}$, C_a^- goto $\{j_1, j_2\}$, if $C_a = 0$ goto j_1 else goto j_2 ,

where $a \in \{l, m, r\}$, $0 \leq j_1, j_2 < n$.

- ▶ **configuration**: tuple (i, j, k, l) , where $0 \leq i < n$ is the next instruction to be executed, and $j, k, l \in \mathbb{N}$ are the current values of the counters C_l , C_m and C_r .
- ▶ **computation**: infinite sequence of consecutive configurations starting from the initial configuration $(0, 0, 0, 0)$.
- ▶ computation **b -recurring** if the instruction labelled b occurs infinitely often in it.
- ▶ computation is **lossy** if the counter values can non-deterministically decrease

Theorem (Alur & Henzinger 1994, Schnoebelen 2010)

Deciding whether a given non-deterministic 3-counter machine has a (lossy) b -recurring computation for a given b is $(\Sigma_1^0\text{-complete})$ $\Sigma_1^1\text{-complete}$.

Undecidability results

Theorem

Model checking for $\text{TeamLTL}(\emptyset, \subseteq)$ is Σ_0^1 -hard.

Model checking for $\text{TeamLTL}(\emptyset, \subseteq, A)$ is Σ_1^1 -hard.

Proof Idea:

- ▶ reduce existence of b -recurring computation of given 3-counter machine M and instruction label b to model checking problem of $\text{TeamLTL}(\emptyset, \subseteq, A)$
- ▶ $\text{TeamLTL}(\emptyset, \subseteq)$ suffices to enforce lossy computation
- ▶ $(T[i, \infty], 0)$ encodes the value of counters of the i th configuration
the value of C_a is the cardinality of the set $\{t \in T[i, \infty] \mid t[0](c_a) = 1\}$

Conclusion

- ▶ TeamLTL is a promising purely modal alternative for a logic for hyperproperties
- ▶ Expressiveness
 - ▶ Uncomparable with HyperLTL
 - ▶ Assuming left-flatness and downward closure translates to $\exists_q^u \forall_\pi \text{HyperQPTL}$.
 - ▶ In general translates to HyperQPTL^+ .
- ▶ Complexity
 - ▶ In EXPSPACE assuming left-flatness and downward closure
 - ▶ In EXPSPACE assuming k-coherence
 - ▶ $\text{TeamLTL}(\subseteq, \otimes)$ already undecidable
 - ▶ $\text{TeamLTL}(\subseteq, \otimes, A)$ highly undecidable

Conclusion

- ▶ TeamLTL is a promising purely modal alternative for a logic for hyperproperties
- ▶ Expressiveness
 - ▶ Uncomparable with HyperLTL
 - ▶ Assuming left-flatness and downward closure translates to $\exists_q^u \forall_\pi \text{HyperQPTL}$.
 - ▶ In general translates to HyperQPTL^+ .
- ▶ Complexity
 - ▶ In EXPSPACE assuming left-flatness and downward closure
 - ▶ In EXPSPACE assuming k-coherence
 - ▶ $\text{TeamLTL}(\subseteq, \otimes)$ already undecidable
 - ▶ $\text{TeamLTL}(\subseteq, \otimes, A)$ highly undecidable

Thank you!